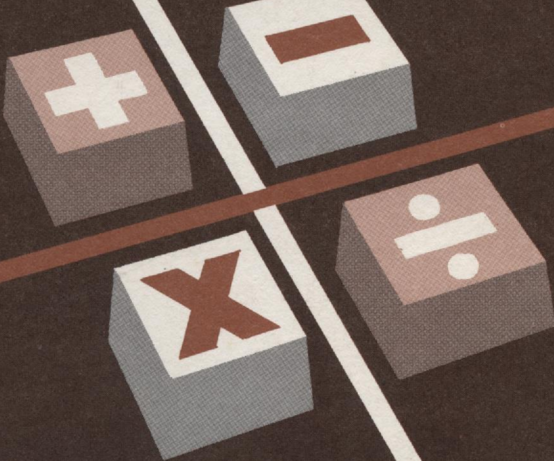
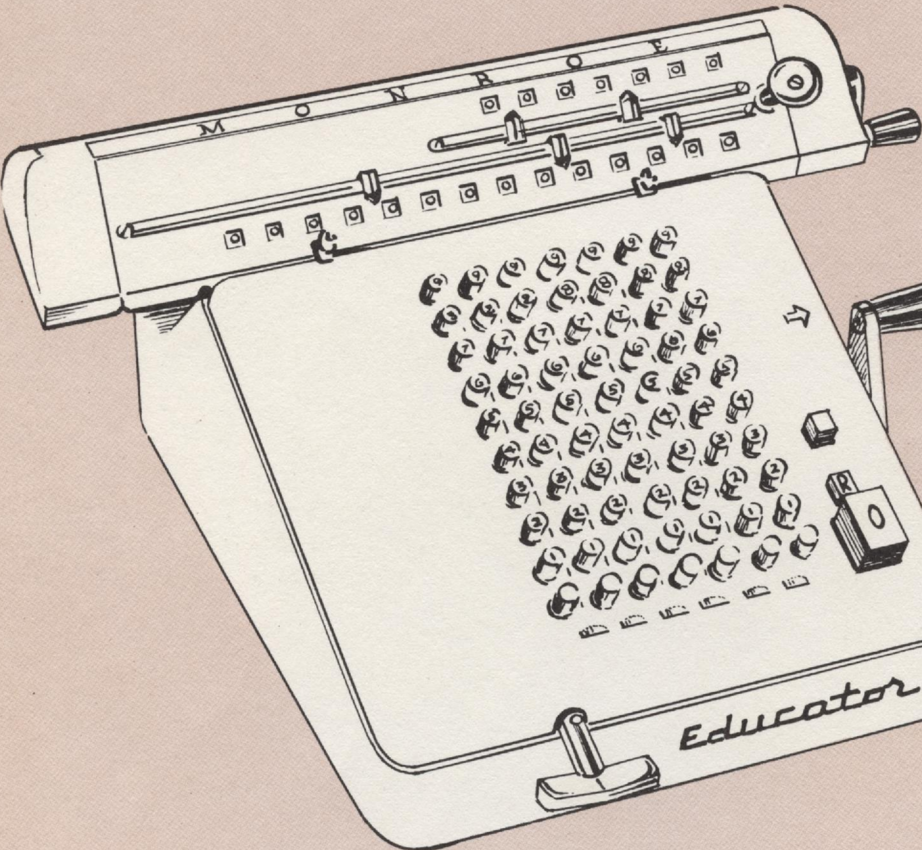


arithmetic minus mystery

= understanding


MONROE
Educator

arithmetic minus mystery = understanding

a better way to teach arithmetic
—with the Monroe Educator
Calculating Machine

MONROE CALCULATING MACHINE COMPANY, INC.
General Offices · Orange, New Jersey
Offices for Sales and Service Throughout the World

Printed in U.S.A.
B-251

a word in advance . . .

Before we can consider clearly the use of calculating machines in the teaching of arithmetic, we must have a clear concept of what arithmetic is as an organized structure of knowledge and the purpose of learning this knowledge. Therefore, we will consider first the nature of arithmetic as a pure science and the possible procedures for acquiring this knowledge. Then it will be possible to relate the Monroe Educator Calculating Machine with regard to its construction and operation to the learning of arithmetic.

WHAT IS ARITHMETIC?

At the start we must recognize that arithmetic, as a mathematical science, has not one, but several modes of construction. As mathematics, all of these constructions deal with abstract entities (numbers) which are related by certain fundamental laws. Of course, in the elementary school the learning does not begin with the undefined abstract elements but with concrete physical models of these elements. The difference in the mathematical theories lies in the way we conceive of these entities or fundamental elements.

One concept of arithmetic holds that the numbers 1, 2, 3, etc. exist as complete entities in themselves and one does not construct them. One only finds the relations that exist between them. When one has discovered or learned all these relations, he has learned the arithmetic of the integers. In other words, each number is conceived fundamentally as a whole, as an entity, and not as made up of ones. One discovers certain things about these numbers which lead to an order (by size), relations (by sums, factors, products or the multiplication table), operations, laws and a science.

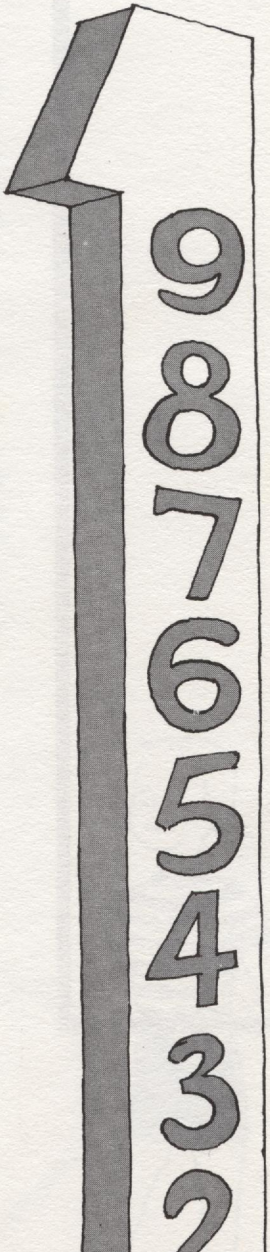
Another approach to the structure of arithmetic is one based on counting, or on adding "one". One literally constructs his numbers, and the way he constructs them leads to certain relations between the numbers. Thus 10 is the successor of 9, i.e. it is 9 plus 1.

This concept is the one almost universally held in the teaching of meaningful arithmetic in the U.S.A. Of course, both structures are meaningful if presented in the proper manner. However, the idea of counting and in this way creating numbers, of numbers being composed of individualized elements, appears to be the desired approach in our country. The Monroe Educator Calculator, which is essentially an adding machine and even in essence a machine for adding "one", is ideally adapted to the teaching of this structure of arithmetic.

THE COMMON STRUCTURE OF ARITHMETIC

Whether we conceive of 5 as an entity in itself, or as the size of a set of distinct elements all belonging to the same collection, the structure of arithmetic has the same form. The size of a number or the order of size of entities leads to an order of the numbers which results in counting. When entities or sets become too numerous, we break them into parts. This results in a system of numeration and the common system today is the decimal positional system. Just as we have individual marks or characters to represent the numbers seven, eight, and nine, we could as well have additional distinct characters to represent ten, eleven, twelve, etc. It is immediately apparent that such a system would become impossibly unwieldy, and so the decimal positional system has been devised. In this system the mark "1" can mean one, ten, one hundred, one thousand, etc., depending on the position in which it appears.

The point we stress here is that having developed a set of numbers and a decimal positional system for putting these numbers in order, we should henceforth capitalize on this knowledge in building the rest of our arithmetical knowledge. Arithmetic is to be conceived as a structure, something built on a previous foundation, and not as a myriad of isolated facts scattered all over the field.



THE LAWS OF ARITHMETIC

ADDITION

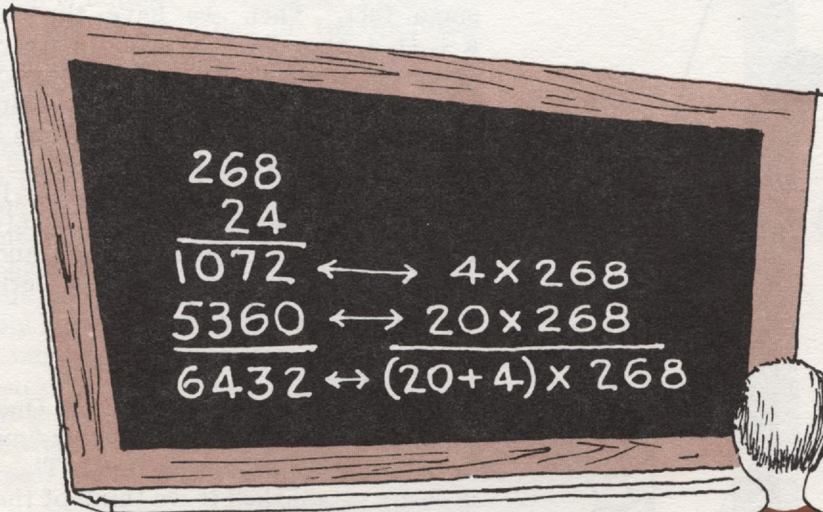
In building this structure, certain laws are essential. First we know that in addition, $a + b = b + a$. Because of this law, we do not teach $7 + 2 = 9$ and $2 + 7 = 9$ as separate and distinct facts. They are the same fact. Then we have the law that $a + (b + c) = (a + b) + c$. In other words, the order in which we add numbers does not affect the result. This law also helps build facts, especially when we get beyond sums of 10. When we write the number 12 in our decimal system, we are indicating $10 + 2$. Thus, $12 + 12$ equals $(10 + 2) + (10 + 2)$. However, in our common pencil and paper method, the sum $12 + 12$ is derived by adding $(2 + 2) + (10 + 10)$.

MULTIPLICATION

Multiplication often gives trouble. One reason is the use of the word "times" or "multiply" in defining the operation itself. The concept is easily established if we think of the operation 5×6 as being the combination of five sets each of which contains six elements. Thus, it is easily seen that multiplication is merely an extension of the process of addition which has already been learned. Once the concept has been grasped, it is easy enough to develop the law $a \times b = b \times a$. The child must use this law to get a real feeling for arithmetic, and it is a great aid in teaching the facts.

The law $a \times (b \times c) = (a \times b) \times c$ is equally important in learning to multiply by 20, 30, or higher decades. Since $20 = 10 \times 2$, $20 \times 9 = (10 \times 2) \times 9$. However, when we perform this operation, what we actually do is to multiply 2×9 and then multiply the result by 10, symbolically $10 \times (2 \times 9)$.

If one is to understand the usual procedures in multiplication, the law $a \times (b + c) = a \times b + a \times c$ is essential. Once this has been developed, the usual algorism can be illustrated as follows:



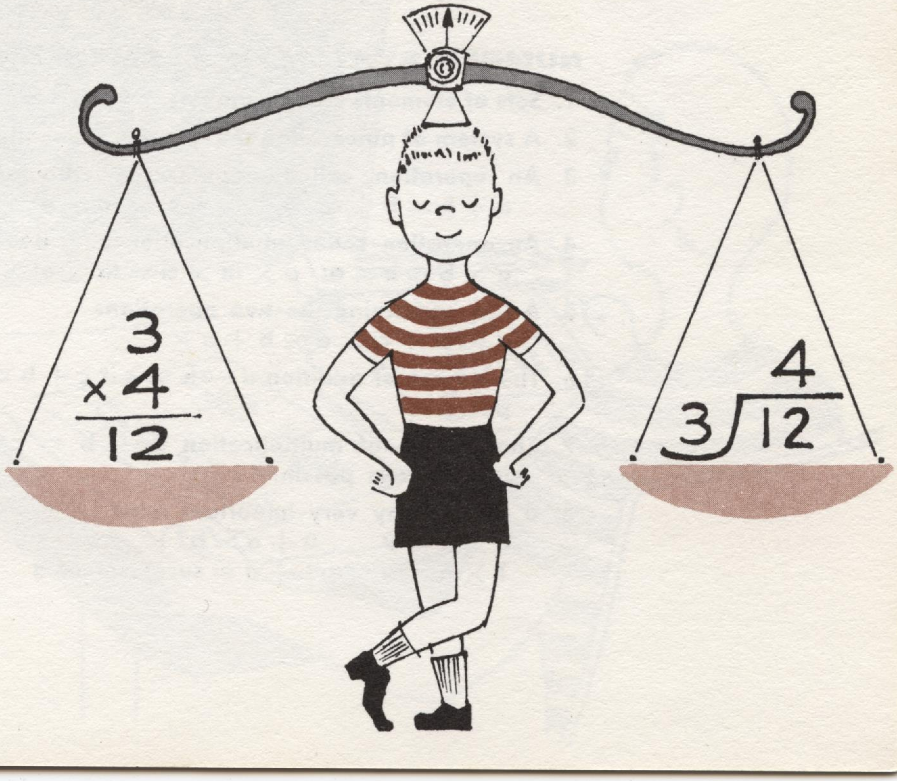
SUBTRACTION

To complete the structure of arithmetic we must consider the inverses of the fundamental operations. Subtraction is merely taking one of the addends (known) from a known sum. Thus a number which was added to another number to get a sum is now drawn out of the sum, and the result is called a difference. It is essential, very essential, that children learn the fact that this operation is not always possible. $6 - 2$ is 4, but $2 - 6$ is impossible in the school arithmetic.

DIVISION

Finally we have the operation of division. There has been so much verbiage about this operation in the past ten years, that many teachers, pupils and even writers have achieved a complete state of confusion. Such words as "measurement", "sharing", "dealing out equally", etc., might well be abandoned. Division is the inverse operation of multiplication. Given a number (dividend), find equal numbers whose sum is the same as that number. If the size of these equal numbers is known, find the number (originally the multiplier) of them, or if the number of them is known, find their size (originally the multiplicand). This is the simplest way and one directly applicable to all physical situations.

There is just one note here. Division is not always possible. When there are remainders we treat them intelligently. Either we discard them, or if fractions have been taught we make further subdivisions. But, even without remainders, division by zero is not possible. We must show why, and this is quite easy to do if division is taught as the inverse of multiplication.



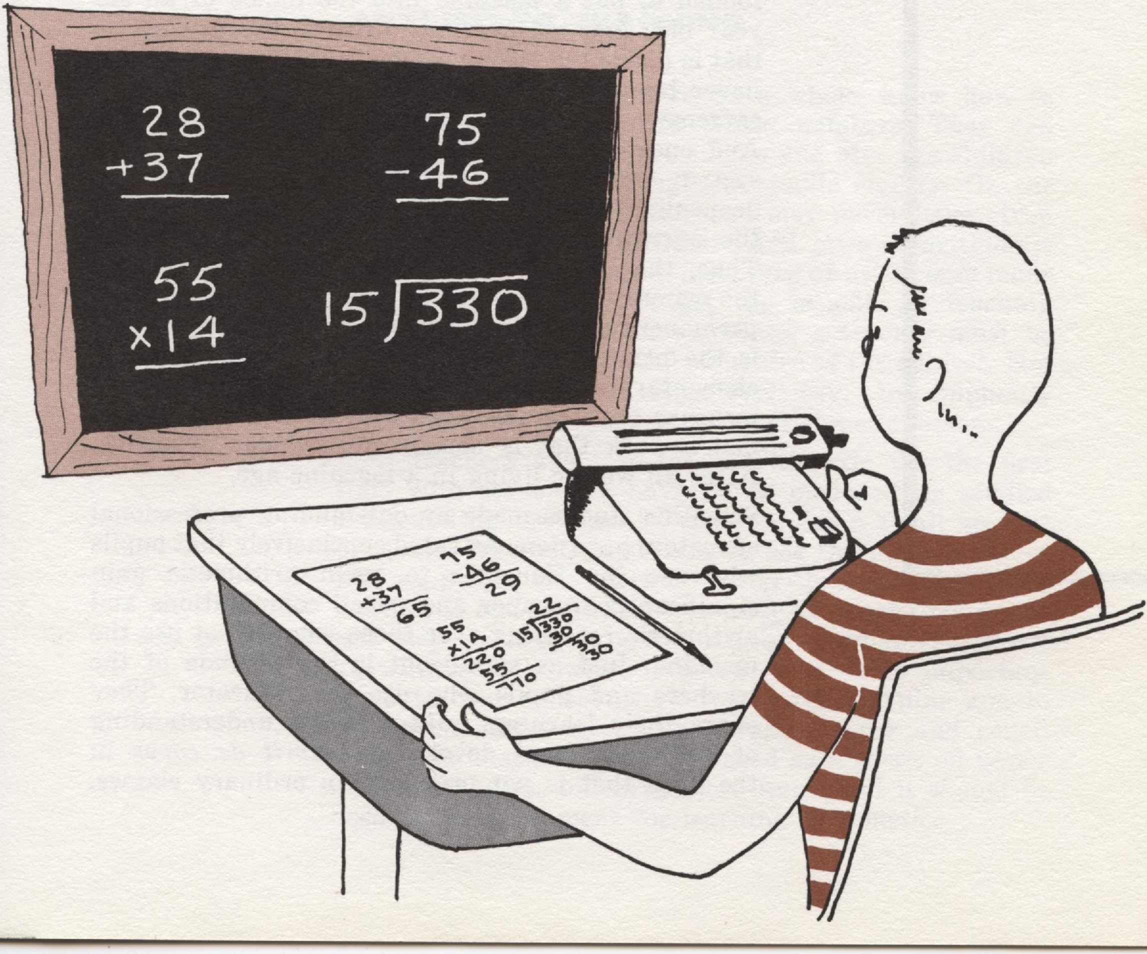
Of course, fractions are also taught in the elementary schools. But these numbers are an entirely different number system from the whole numbers and should be taught as such. Since they are different, we know that there will be a different rule for addition and a different rule for multiplication, but the *laws* of the operations still hold. This is what gives structure to arithmetic. Further these fractions now make it possible to always divide (except by zero) and this is the only reason for teaching fractions, both from a pure and an applied point of view. The Monroe Educator is a decimal place system computing machine and hence does not treat the theory of common fractions. It is capable of treating the system of decimal fractions, but these fractions are limited in scope, in that many common fractions are incapable of finite decimal fraction representation. The machine is essentially a whole number tool. Thus it is essentially used to teach the following:

NUMBER SYSTEM OF ELEMENTARY SCHOOL

1. **Sets of elements called numbers**
2. **A system of numeration to represent all numbers (Decimal)**
3. **An operation called addition (+), bound by the laws**
   $a + b = b + a; \quad a + (b + c) = (a + b) + c$
4. **An operation called multiplication (×), bound by the laws**
   $a \times b = b \times a; \quad a \times (b \times c) = (a \times b) \times c$
5. **A law combining the two operations**
   $a \times (b + c) = a \times b + a \times c$
6. **The inverse of addition $a - b = c$ if $c + b = a$ (not always possible)**
7. **The inverse of multiplication $a \div b = c$ if $c \times b = a$ (not always possible)**
8. **0 and 1 play very important roles**
   $0 \times a = 0 \qquad 0 + a = a$
   $1 \times a = a \qquad 1 + a =$ successor of a

METHODS OF LEARNING ARITHMETIC

There are many ways of learning this structure. These have been dwelt on at length in the literature. In general an abstract or rote approach (they need not be the same, however) is not advocated for learning arithmetic in the elementary school. The method advocated by most educators and psychologists is to start with a physical system and pull out of this system those particular qualities called number, and then organize these qualities into arithmetic. For obvious reasons your teaching will be better with specifically designed materials which appeal to youngsters.



THE NATURE OF THE EDUCATOR CALCULATOR

The entire structure of arithmetic just reviewed is built into the Monroe Educator. Now it would be foolish to put a machine into the hands of all six year olds and tell them to discover the arithmetic that is inside the machine. Obviously the teacher will never be *replaced* by machines. But the machine is extremely helpful in teaching the number system. And once children have learned about the number system, the machine serves as an instrument for deepening the understanding and tying together all the operations and laws into a unified structure. Thus, the Educator is extremely valuable at both the primary and secondary stages of learning to give permanency, depth and structure to arithmetic. This is the prime purpose of learning arithmetic in the elementary school. But it also shows in no small measure the value of the machine in doing part of man's work that is usually an arduous drudgery. After all we are living in a machine age.

Scientific studies made by outstanding professional educators have demonstrated conclusively that pupils who use the Educator to learn arithmetic gain significantly in paper and pencil computations and arithmetic reasoning over those who do not use the machine. Just as significant is the attitude of the teachers and pupils who use the Educator. They enjoy their learning, gain a better understanding of arithmetic, and develop an *esprit de corps* in the class that is not prevalent in ordinary classes.

HOW THE EDUCATOR IS APPLIED TO TEACHING

Elementary school teachers, as a whole, know how to teach the operations with whole numbers. They can tell children how to do arithmetic and give the children a fairly good idea of what the results signify. On the other hand, these same teachers may never have given much thought to the structure and laws of arithmetic and especially the internal structure of arithmetic made possible by its laws. To get the benefits of teaching arithmetic by the use of machines, a teacher must be aware of the mathematical structure of the subject. Not that she teaches the structure explicitly. But implicitly it acts in every procedure in the arithmetic lesson.

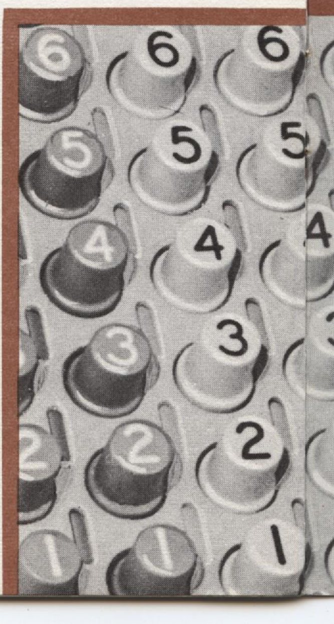
Most teachers about to teach arithmetic for the first time with Educators have never operated or studied a calculating machine. So they require a small amount of special instruction on how to use the machine. Now just as people can learn to do arithmetic with paper and pencil and not know why they do what they do, so can they learn to do arithmetic on a machine without knowing exactly what is being done. It is important for a teacher to learn how to use the machine and to see how machine operation parallels paper and pencil work. The machine should be used as a check on paper and pencil work, but even more important it should be used as an instrument for learning arithmetic.

Here are a few elements of arithmetic for which the Educator can be used with maximum effect:

1. **Counting. A decimal place system. The role of zero.**
2. **Adding. The nature of exchange (carrying).**
3. **Subtracting. The nature of exchange (borrowing).**
4. **Multiplication. As repetitive addition of the same multiplicand. The meaning of $\times$ by 30, 200, etc.**
5. **Division. As repetitive subtraction of a divisor.**
6. **The laws of addition and multiplication, e.g. $(a + b) + c = a + (b + c)$.**
7. **The roles of 0 and 1.**
8. **The decimal fractions.**

Of course, arithmetic can be learned without the use of calculating machines. It has been in the past and for some time to come will continue to be taught as paper and pencil calculation, or with the use of materials and pictures. The machine offers a highly superior method. From a mathematical point of view there are no disadvantages in its use. There is one danger from a pedagogical viewpoint but it is easily avoided if we are on guard against it.

If a teacher were unaware of the theory of arithmetic and the theory of the construction of the Monroe Educator, she might use the machine as a tool to get an answer, and by methods that are good machine methods but poor learning methods. This is readily avoided if the teacher has a clear-cut understanding of the subject and makes a determined effort to teach it properly. Otherwise, the children are no better off than without machines.



There are, however, strong, sound cultural values to be gained by the proper use of the Educator. We can list these as follows:

1. **The Educator clarifies the structure and meaning of arithmetic.**

2. **The Educator has a greater ability to arouse interest in doing arithmetic than any other device. It has tremendous motivation power.**

3. **The Educator by its orderly arrangement, begets orderliness in the paper and pencil work of the pupils.**

4. **The Educator fits into today's world. The pupil is doing what adults are doing in the offices and shops. He feels he is learning something important.**

5. **The Educator is a model. It helps pupils to see other models in the work around them, and hence helps to develop problem-solving ability.**

6. **The Educator helps to improve speed, accuracy and neatness in paper and pencil arithmetic.**

7. **The Educator develops an *esprit de corps* in the classroom, and also acts as a disciplinary (in the sense of learning) tool. A machine does only what you tell it to do.**

With the proper instruction, from conscientious teachers, children will gain far more being taught arithmetic through the use of the Monroe Educator Calculating Machine.